How to weigh a galaxy

Name of institution : Institut d'Astrophysique de Paris

Title of the educational scenario template : KLiC scenario

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Subject

The teaching activities consist in measuring the mass of a spiral galaxy, viewed edge-on, using the same procedure employed by astronomers. It is surprising how just a few measurements and the knowledge of a few fundamental laws of physics make it possible to weigh the largest and most distant objects in the Universe, despite the impossibility of carrying out direct measurements and the fact that the only information available to us derive from a few photons which have travelled for tens and tens of millions of years. What is even more surprising is the fact that the simple measurement proposed here makes it possible to obtain experimental evidence for the existence of the famous dark matter. Dark matter and dark energy are believed to make up to 96% of the total content of the Universe, but they are incredibly difficult to detect and study, other than through the gravitational forces they exert. Investigating the nature of dark matter and dark energy is one of the biggest challenges today in the fields of particle physics and cosmology. The ATLAS and CMS experiments will look for supersymmetric particles to test a likely hypothesis for the make-up of dark matter.

Student Age

17-18 years old and university.

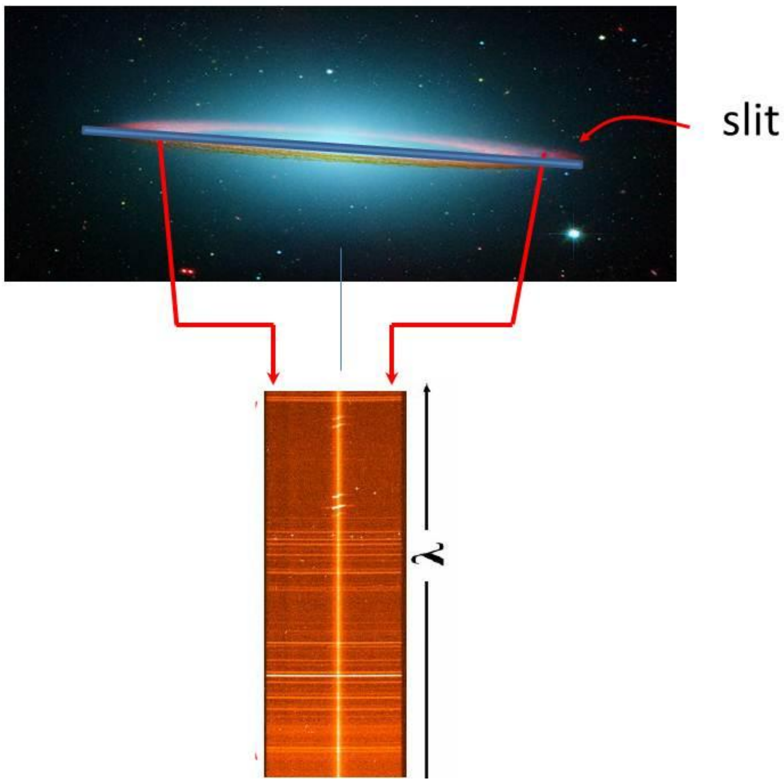
Science content

- Needed science background for the teacher

    - Observation of a galaxy, continuum and emission lines.

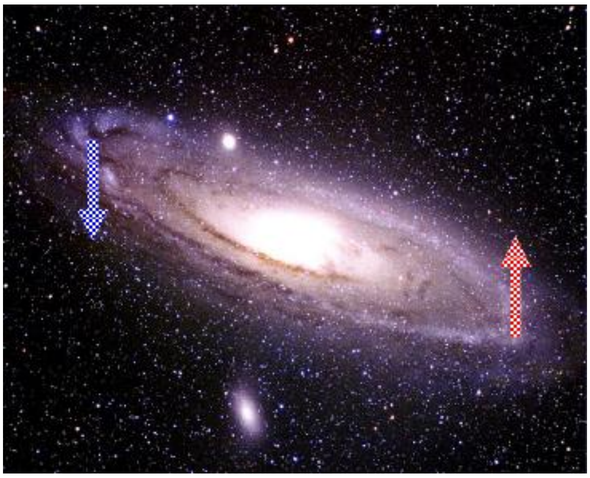
There are two extreme situations when one observes a galaxy. It can be *edge-on* (left figure below; The Sombrero Galaxy) or *face-on* (right figure below; NGC1232).

In the case considered here, the galaxy will be edge-on, and we observe the light along the slit of the spectrograph that goes through the center of the galaxy (cf. image below).



At each position along the slit, the radiation emitted by the matter present in the galaxy is received on Earth and passed through a dispersant element (prism, grating etc.) which separates the wavelengths. In this way, we obtain an image which, along the *x* axis (vertical direction in the figure above), contains the spectrum, i.e. information as a function of the wavelength λ, of the radiation from various points of the slice. In the image of the spectrum, the vertical line at the center represents the emission from the center of the slice, i.e. from the nucleus of the galaxy, which clearly does not just emit lines at specific λ, but a continuum at all the recordable λ's. *In the image that you will download, wavelengths are in the horizontal direction, and position along the slit in the vertical direction*.

- Doppler-Fizeau effect



Spiral galaxies contain a large amount of gas; the gas emits a spectrum of lines; if the galaxy is viewed edge-on (and not directly facing the observer, not perpendicular to the line of sight) and given that it rotates on its own axis, on one side the gas is receding from us and on the other side, with respect to the center, it is approaching us (cf. figure). The lines emitted by a gas which moves with respect to the observer undergo the so-called Doppler effect, which shifts the observed frequency (or wavelength).

The measurement we shall make exploits the fact that the Doppler shift is proportional to the velocity at which the gas moves away or gets closer to us. This radial velocity is directly measurable on the spectrum. We therefore obtain the velocity v of the gas as a function of its distance r from the center of the galaxy. The plot of the velocity v of each point of the galaxy as a function of the distance r is known as the rotation curve. Beyond a given distance, where the matter is not anymore part of the rigid body composed by the nucleus of the galaxy, the velocity is in turn proportional to the mass enclosed in the radius r, as is described by the Kepler's laws.

- Dark matter

When studying galaxies (as done in this scenario) it is invariably found that the rotational velocity remains constant, or "flat", with increasing distance away from the galactic center. This result is highly counterintuitive since, based on Newton's law of gravity, the rotational velocity would steadily decrease for matter further away from the galactic center. Analogously, inner planets within the Solar System travel more quickly about the Sun than do the outer planets (e.g. the Earth travels around the sun at about 100,000 km/hr while Saturn, which is further out, travels at only one third this speed; this is the keplerian rotation). One way to speed up the outer matter within a galaxy would be to add more mass within the halo of this galaxy. This extra-mass is not seen directly on photos. Thus, the flat galactic rotation curves seem to suggest that each galaxy is surrounded by significant amounts of dark matter. It has been postulated, and generally accepted, that the dark matter would have to be located in a massive, roughly spherical halo enshrouding each galaxy.

The existence of dark matter and the search of the particles corresponding to this type of matter is still a controversial subject. You may find more details on the pedagogical pages written by a Professor of Physics at Berkeley, M. white (http://astro.berkeley.edu/~mwhite/welcome.html - current research interests).

- Pre-requisite knowledge for the student: Newton Laws only (the required relation between mass and velocity can also be provided by the teacher).

Learning objectives

The learning activity proposed is significant because it favors creatively the pupils' knowledge concerning the concepts of: movement, acceleration, light, and also, offering to the pupils the opportunity to test their own scientific hypothesis and also to test these hypothesis' applicability in new learning contexts.

At the end of the learning activity the pupils will be able to improve and develop the competencies of:

- communication and interpersonal relationship
- identifying variables
- constructing a table of data and a graph
- hypothesizing and elaboration of testing strategies for these hypothesis

Inquiry-based character

- The students explain possible solutions
- The teacher gives new definition and expects students to use them
- The students infer new relations and apply them on data

Applied technology

- SalsaJ software
- Google Sky

Discussion guide

1. Physical description of the galaxy NGC7083
    a) Use Google Sky to locate the galaxy
    b) Look for its distance and inclination
    You should find that this Galaxy is at a distance of 39.7 Mpc to Earth (1pc = 3,09. 10^{16} m) and is inclined by 30 degrees with respect to the line of sight from the Earth.
2. How to relate the mass M of the galaxy and the observed spectrum?
    1. Preliminary discussion on galaxy and spectrum.
        a. Different type of galaxy
        b. Different spectrum and type of lines
        c. Link between lines and nucleus: **nuclear physics** provides us with accurate determination of emission lines.

    2. Determine the relation between the total mass of the galaxy M and the velocity of a nucleus of mass m in rotation on a circle of radius r from the center of the galaxy.
    There are actually two relations ! Either for a nucleus located in the galaxy, acting as a rigid body with angular speed, ϖ : $v = \varpi r$; or for a nucleus of mass m, outside the galaxy and subject to the gravity force of the galaxy.
        a. You may either provide the relation and discuss it or try to infer it.
        b. Students may discuss which physical parameters should be involved in this relation, and then use units to determine both cases and relations.
        c. Most students should be able to derive the first relation.

d. Advanced students may derive the second relation,
   i. assume a constant radius
   ii. then, potential energy is constant
   iii. then, kinetic energy is constant, so v=Cte
   iv. in the polar coordinates, with R=cte and v=cte, one derives the acceleration: $a = \frac{v^2}{r}$
   v. the Newton laws yield the relation

$$\frac{GmM}{r^2} = \frac{mv^2}{r}$$

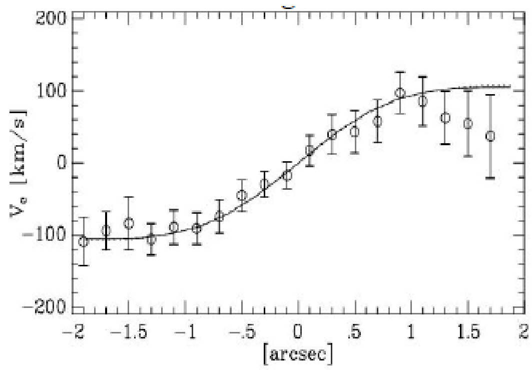
3. Relation between the observed wavelength of one line (emitted by the satellite at a distance r) and the velocity of this satellite.
   a. Connection with sound (sound = wave)
   b. Generalization to the Doppler shift (you may open the discussion about the Hubble flow of the Universe, which was first observed in the shift of emission lines of distant galaxies).
   c. **The teacher has to point out here that the velocity of the nucleus as observed by us could be zero if the galaxy was observed face-on…**
   d. The angle between the galaxy and the line of sight modifies the projection of the velocity along the line of sight:
   $V_{longitudinal} = V_{rotation}\cos(2\pi - i) = V_{rotation}\sin(i)$
   We search $v = V_{rotation}$ and the Doppler shift gives $V_{longitudinal}$

   Finally, $\frac{\Delta\lambda}{\lambda} = \frac{\mathbf{v}\sin(\mathbf{i})}{c}$

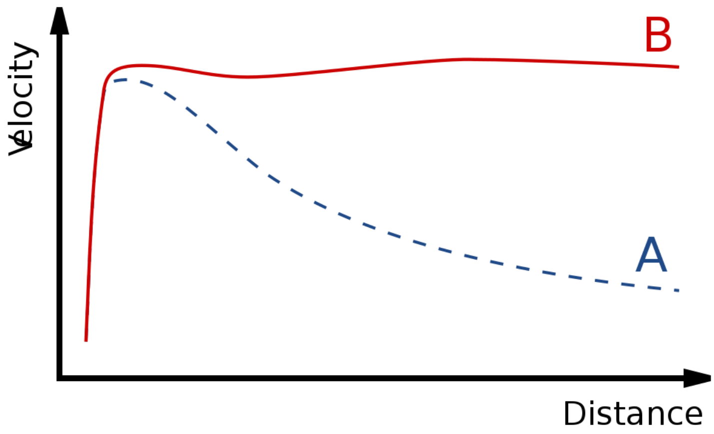
3. Data analysis: going from the spectrum back to the mass of the galaxy
   a) Calibration of the image (students need to realize that this calibration is required – so you may do this step after the next step has been tried!)
      i. Determination of correspondence horizontal pixel <-> wavelength (using the doublet NII observed in the galaxy with λ_1 = 6616 Å and λ_2 = 6652 Å, one finds 22 pixels = 20 Å)
      ii. Determination of correspondence vertical pixel <-> radius
      Numbers given to the students
         The galaxy is at R=39.7 Mpc (1pc = 3,09. 10^{16} m)
         The angular size of the image = 4 arcmin
      Relation to be derived:
         1 vertical pixel = 0,82 arcsec (= dq)
            This information is given in SalsaJ (Image → Show Info…)
         1 vertical pixel = R dq = 4,88.10^{18} m (with dq in radians)
   b) Determination of $\Delta\lambda(r)$
   c) Determination of $v(r)$

4. Discussion
   a) Is this profile realistic / consistent with your expectations/predictions?

You should find a symmetric profile.



Below, the predicted (blue) and observed (red) curves are compared. Close to the center, the curve is consistent with a rigid body. But further away, the prediction of the evolution of the velocity outside the galaxy (so for a constant mass M) is $\propto \frac{1}{\sqrt{r}}$. This is not consistent with the observed curve.



b) Can you derive the mass M ?

As long as the nucleus is part of the rigid body, the velocity is not linked to the mass but only to the rotational velocity of the galaxy as a whole.

When the matter is outside the rigid body and acts as a satellite, the rotation speed satisfies $v^2=GM/r$, and we can infer the mass *M(r),* that is the mass within radius *r*. Since our curve shows that the velocity is constant with r, it means that the mass must continue to increase proportionally to *r.* **This mass is not seen as luminous matter.** This rise appears to stop at about 50kpc (but you do not see it with the data in hand), where halos appear to be truncated. Astronomers infer that the mass-to-luminosity ratio of the galaxy, including its disk halo, is about 5 times larger than estimated for the luminous inner region.

At some point, the teacher will introduce the notion of **dark matter**… Note that this notion is still theoretical since no **particles** or whatsoever ingredients of the standard model of the physics correspond to this dark matter up to now.

c) Can you derive the mass (r) ?

We can derive the mass at the distance where the profile is not anymore consistent with a rigid body. You should find a **typical result** of 10^{40} kg (at a radius r_0=10 pixels=8 arcsec, and a velocity v=180 km/s). Then the mass (up to a few kpc) is: $10^{40}\ \frac{r}{r_0}$ kg.

To go further:

A. estimates of error bars
   1. Estimate errors in the calibration
   2. How does this error translate in wavelength, in velocity, and finally in mass.

B. Compare to other masses
   1. Estimate the mass of the Sun using the rotation of the Earth, or another planet. There is no indication of dark matter inside the galaxy, and the rotation curve of planets follows the Keplerian laws.
   2. Estimate the mass of the Earth using the rotation of the Moon.

C. Search information about dark matter models and searches (LHC experiments for example)

<u>Assessment</u>

STUDENT WORKSHEET

Unit Title

How to weight a galaxy?

Introduction

The teaching activities consist in measuring the mass of a spiral galaxy, viewed edge-on, using the same procedure employed by astronomers. You are going to use only the spectroscopic image of the galaxy **NGC7083 that can be analyzed with the software SalsaJ.**

You must learn

- **how to read a spectroscopic image**
- **how to interpret spectroscopic data in terms of physical quantities**

Materials

- One image called **NGC7083_RID.FTS**

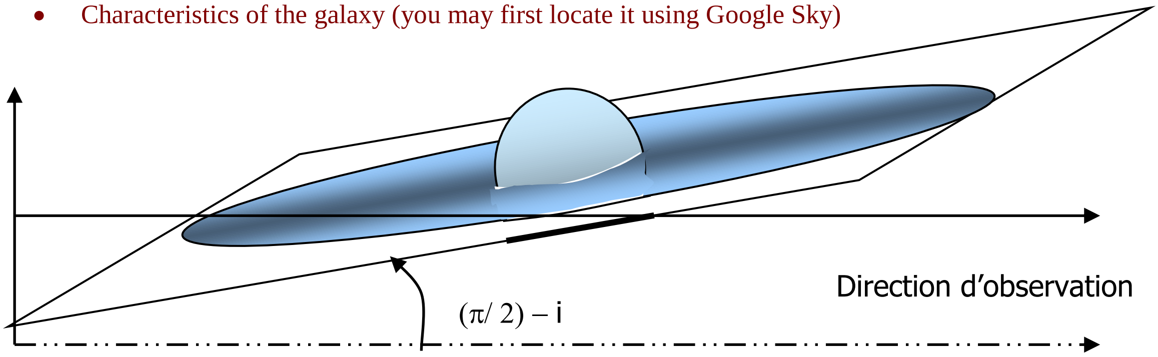
The size of this image is 386x294 pixels.

The vertical axis corresponds to a line going through the galaxy. The relevant units is the angular separation between the two sides of the image, $\Delta\theta$ = 4 arcmin.

The horizontal axis gives the spectrum of the light received on Earth from the material located at each position along the vertical axis. The central horizontal line corresponds to the continuum emitted by the center of the galaxy.

Vertical lines correspond to specific emission lines emitted by matter located at different radius along the galaxy.

- The software SalsaJ.
- Characteristics of the galaxy (you may first locate it using Google Sky)



The distance from Earth to the galaxy is R=39.7 Mpc (1pc = 3,09. 10^{16} m)
The angular size of the image = 4 arcmin
The angular orientation of the galaxy is sin(i) =0.5

Investigation

Observation worksheet

Name and surname of the participants at the experiment:

Category: □pupil; □ teacher student; □ teacher

Age: ____________, Gender: □ M, □ F

A. How to relate the mass M of the galaxy and the observed image?

Hypothesis - Discuss the different physical quantities that can be used to describe the matter within the galaxy. Try to describe how the mass of the galaxy could be linked to some properties of the light that is emitted from the galaxy.

Infer one or more relation(s) between the mass of the galaxy M, its rotational velocity ω, the mass *m* of a nucleus in rotation around the center of the galaxy with a velocity *v*, and the radius of this circle *r* (you may distinguish two cases according to the physical link between the nucleus and the galaxy).

Infer one or more relation(s) between the velocity *v* of a nucleus, and the wavelength of the light that it emits and that we receive on Earth.

How do those relations change if the galaxy makes an angle i with the line of sight to Earth ?

B. Data analysis: going from the spectrum back to the mass of the galaxy

1. Determination of $\Delta\lambda(r)$. Explain how you are going to derive the shift in wavelength as a function of the distance to the center of the galaxy.

Make a plot of $\Delta\lambda(r)$.

2. Determination of $v(r)$. Explain how you are going to derive the rotational velocity as a function of the distance to the center of the galaxy.

Make a plot of $v(r)$.

C. Discussion
   1. Is the profile of the velocity realistic/consistent with your expectations/predictions?
   2. Can you answer the initial question and tell something about the mass?