Space travel – some mathematics behind

Name of your Institution: Universidad Complutense de Madrid/AEGORA

Title of the educational scenario template: Inquiry-based teaching

Title of your educational scenario: The space traveler – some mathematics behind

Educational problem

At school students are taught Mathematics to solve well defined problems. They perceive Mathematics as safe since once the sample problem is taught during the lecture, all similar problems are solvable in the same manner. However, Mathematics are also about research. There are unsolved mathematical problems and some of them are related with everyday life. Among those problems, there is one that can be easily posed to mid/high school students: the "traveler problem". In this case, the space traveler problem.

The "*space traveler problem*" is a classic optimization problem; students are asked to visit many planetary systems and they have to define the optimal path: the shortest one. In order to do that, they'd have to compute all the possible paths and select the best. This would require enumerating all possible permutations. They will find out that this is easy to do but takes a lot of time (!). In fact, it may take such a long time that the problems could not be solved. Then, the concept of heuristic algorithms defined to find good solutions for problems that cannot be solved exactly is naturally introduced. The objective of this scenario is to familiarize students with mathematical research.

Educational scenario objectives

During this scenario, students will

1. learn about the main planes used as reference for astrography: the equator and the galactic plane.

2. learn the actual location of planetary systems in Space, whether they are clustered or not.

3. learn about measuring distances and transforming between Cartesian and spherical coordinates.

4. learn about the "*space traveler problem*" and some simple heuristics to find good solutions. In this manner, they are introduced in concepts like artificial intelligence.

Characteristics and needs of students

Students have limited exposure to mathematical research. Mathematics are taught like a language or a tool for other subjects: Physics, Chemistry, Technical Design. Students tend to reject Mathematics as a subject in mid/high school because the language is abstract. This exercise provides an appealing environment to excite the mathematical thinking and discussion in an open mathematical problem.

This exercise will allow students to interact (e.g. by working in pairs) and develop social and collaboration skills, allowing them to see that Science can be a group activity and not only a solitary one. This change of perception may trigger an increased interest in Science in many of them, and possibly a turn to Science careers.

Rationale of the Educational approach and Parameters guaranteeing its implementation

The activity is designed according to the Inquiry-Based model and it follows a scientific approach. Students are asked to make predictions on the optimal path to visit several planets. Based on their research, they are asked to come up with their own conclusions and propose an optimal solution. Different groups are expected to produce different trips; an open discussion among them will help in maturing the knowledge.

Learning activities

1. Question-eliciting activities

a. exhibit curiosity

You may begin your lesson with some videos describing the origins of the Chess game. Stories about the invention of chess are not vary. However, they all incorporate the same geometric progression problem.

[(http://en.wikipedia.org/wiki/Wheat_and_chessboard_problem]

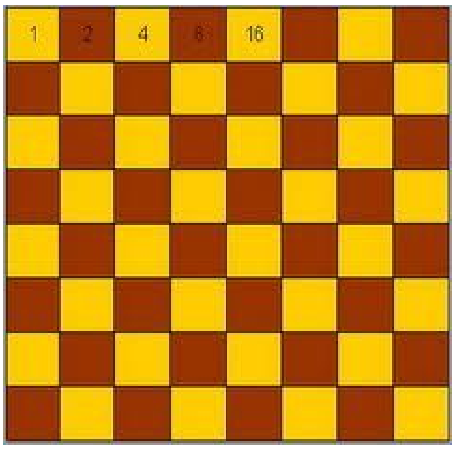

Figure 1: The creator of the game of chess gave his invention to the King who has recently lost his son. The King was so pleased as to gave the inventor the right to name his prize for the invention. The man asked the King the following: for the first square of

the chess board, he would receive one grain of wheat (in some tellings, rice); two for the second one; four on the third one... and so on, doubling the amount of wheat each time. The ruler, arithmetically unaware, quickly accepted the inventor's offer, to later realize that he could not fullfill his compromise.

This exercise can be used to demonstrate how quickly exponential sequences grow, as well as to introduce exponents, zero power and geometric series.

Then present the space traveler problem using the applet developed by UCM/AEGORA (see below). 'The Stellar Traveler' tool is available within the wiki based service in Hands-On Universe (Spain) [url: www.houspain.com]. The tool is programmed as a JAVA applet and requires to have JAVA installed. It works with any navigator. There, you will find some estimates on permutations and computer time required to evaluate the optimal path to visit the 140 planetary systems implemented in the tool.

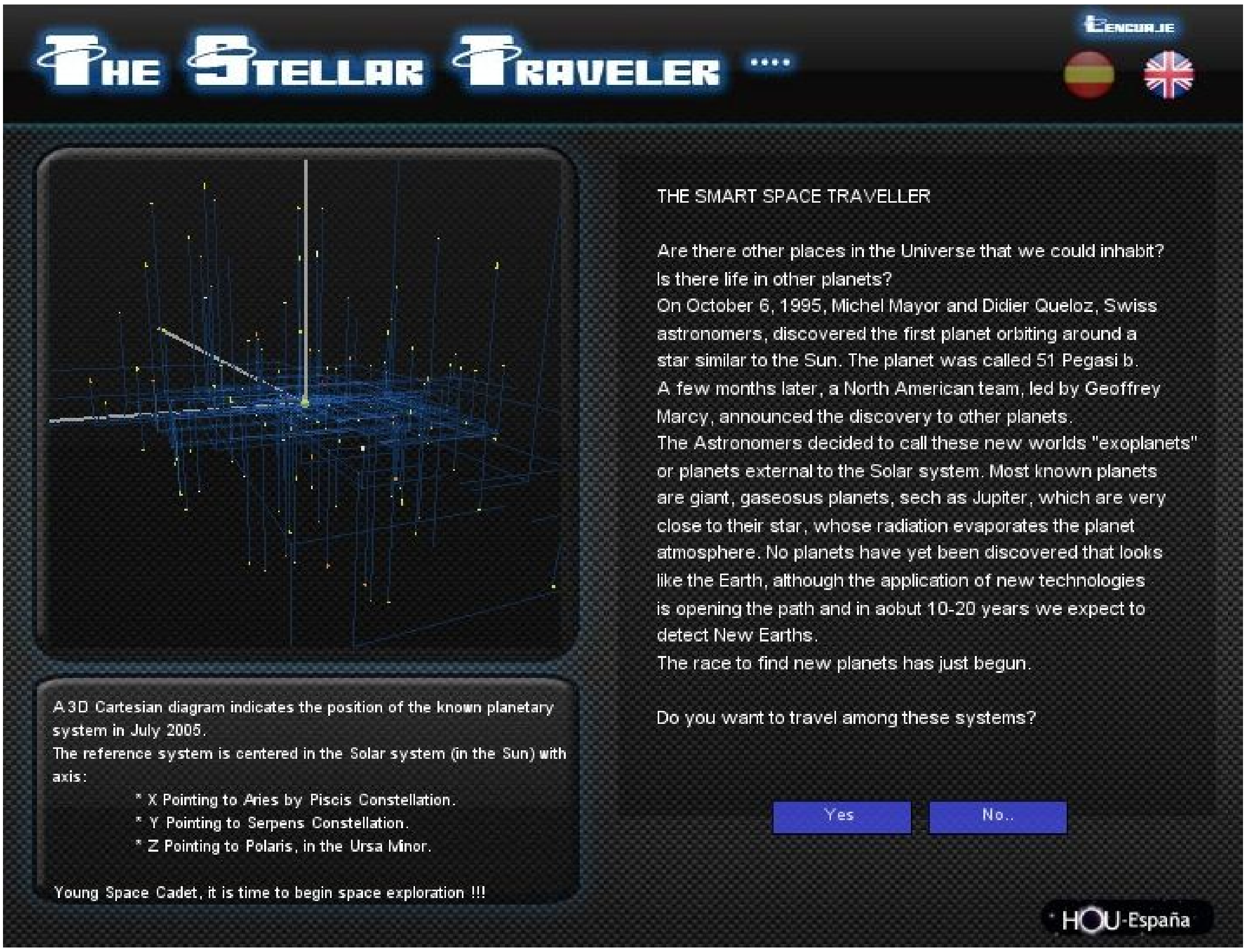

Figure 2: Access page to the tool. An interactive e-learning system provides tips to students on the main characteristics of the systems and the mathematical knowledge require to measure distances and plan an efficient trip.

You may inform your students about what they will do during this exercise:

- Learn about the references in space to locate stars and planetary systems.
- Study the distribution of extra-solar planetary systems and their properties.
- Learn to measure distances in space from astronomical coordinates.
- Find out the time to travel among them.

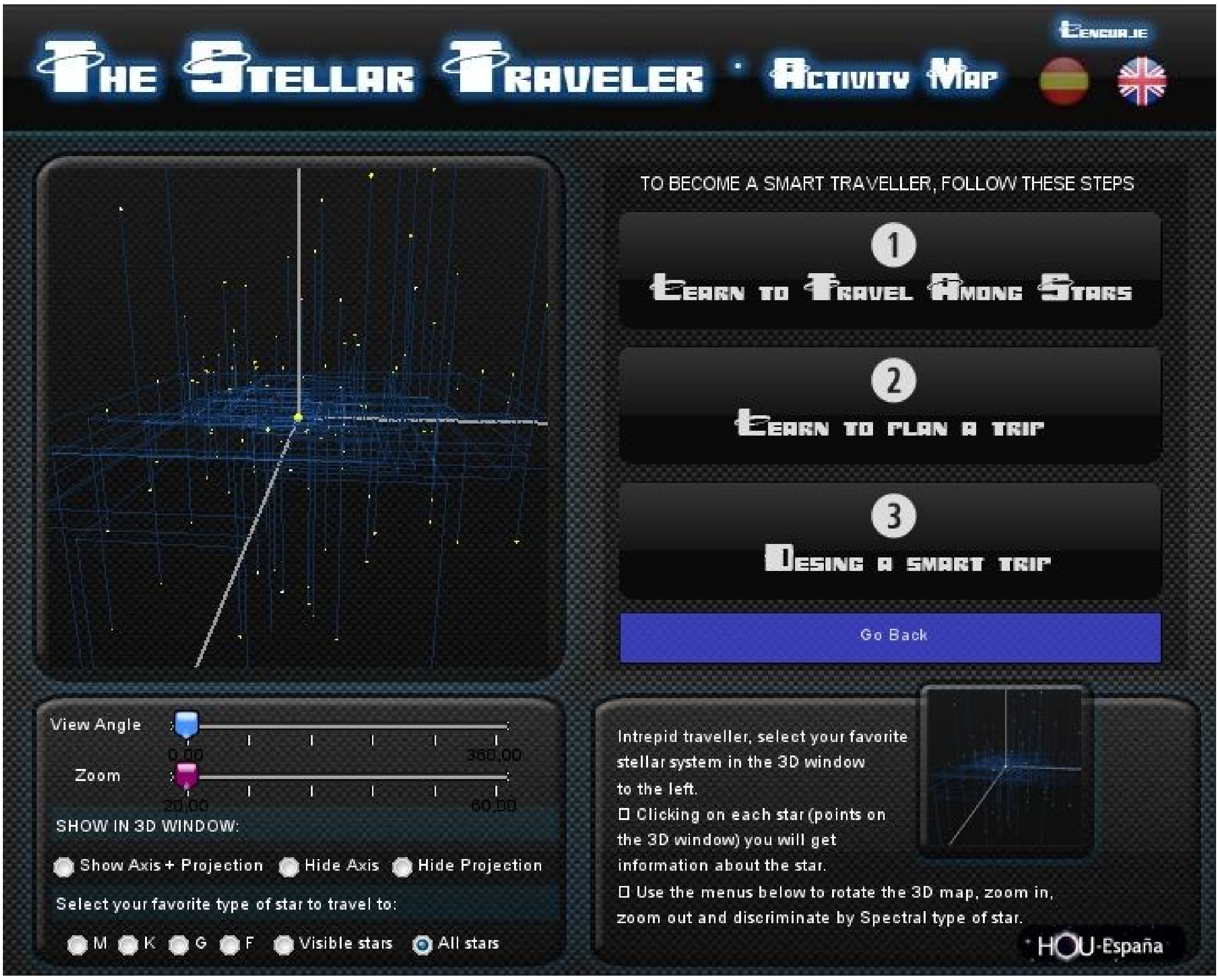

Figure 3: Access page to the three activities implemented in the tool. Activity (1) teach students how to measure distances and transform from spherical (astronomical) coordinates to Cartesian coordinates. Activity (2) introduces to optimization through a simple permutation activity with 4 stars. Activity (3) drives the students into the optimization problem when the numbers increase; this step is done with 10 stars and there is a tool implemented to assists students in the understanding of the nearest neighbor heuristic.

b. Define Question from current knowledge

During your discussion with students make sure to ask them some of the following questions in order to engage them further and check their background regarding the subject.

1. How distances between stars are measured?
2. Use the Java applet to estimate the optimal path when only visiting 4 planetary systems
3. Why it is required to enumerate all possible permutations?
4. Why the number of permutations increases exponentially with the number of planets to visit?
5. Make together a small program computing the permutations of an increasing number of planets.

Finally, trigger conversation with your class by asking your students about permutations of some few items and how do they grow when the number of items increases. Drive them to relate this problem with exponential grow and the chess problem above.

2. Active investigation

a. propose preliminary explanation or hypothesis

Make your students propose strategies to find out the optimal path for several selections of ten planetary systems. Students should select different clusterings of the planetary systems to visit. Also, different groups can study different types of clusterings.

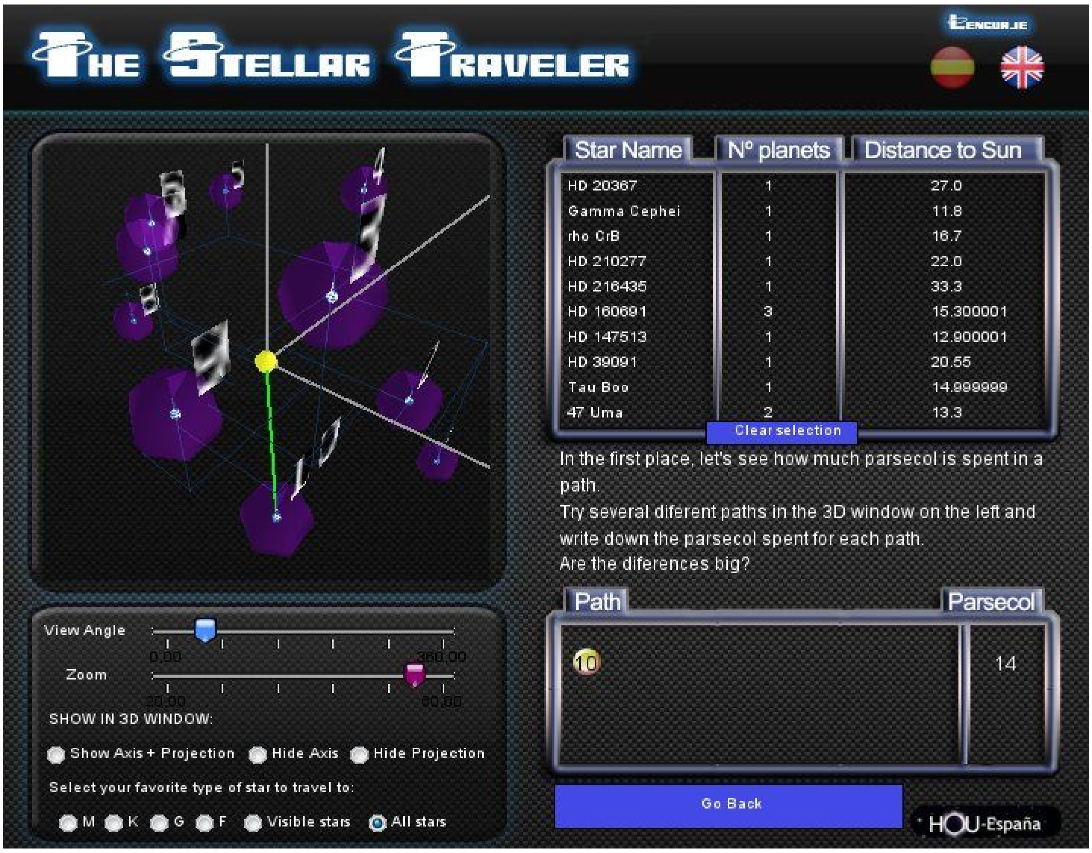

Figure 4: Access the Activity (3). There, you'll find a tool to interactively select from the 3D map, the 10 planetary systems to travel to. Information on distances to the Sun is provided to guide the selection of the optimal path.

b. plan & conduct simple investigation

After students have made their predictions, you may divide your class into working groups.

Propose a simple strategy to find the optimal path: to travel always to the nearest planetary system from your current position. The tool will aid students to define this optimal path making use of a simple, neighborhood search heuristic.

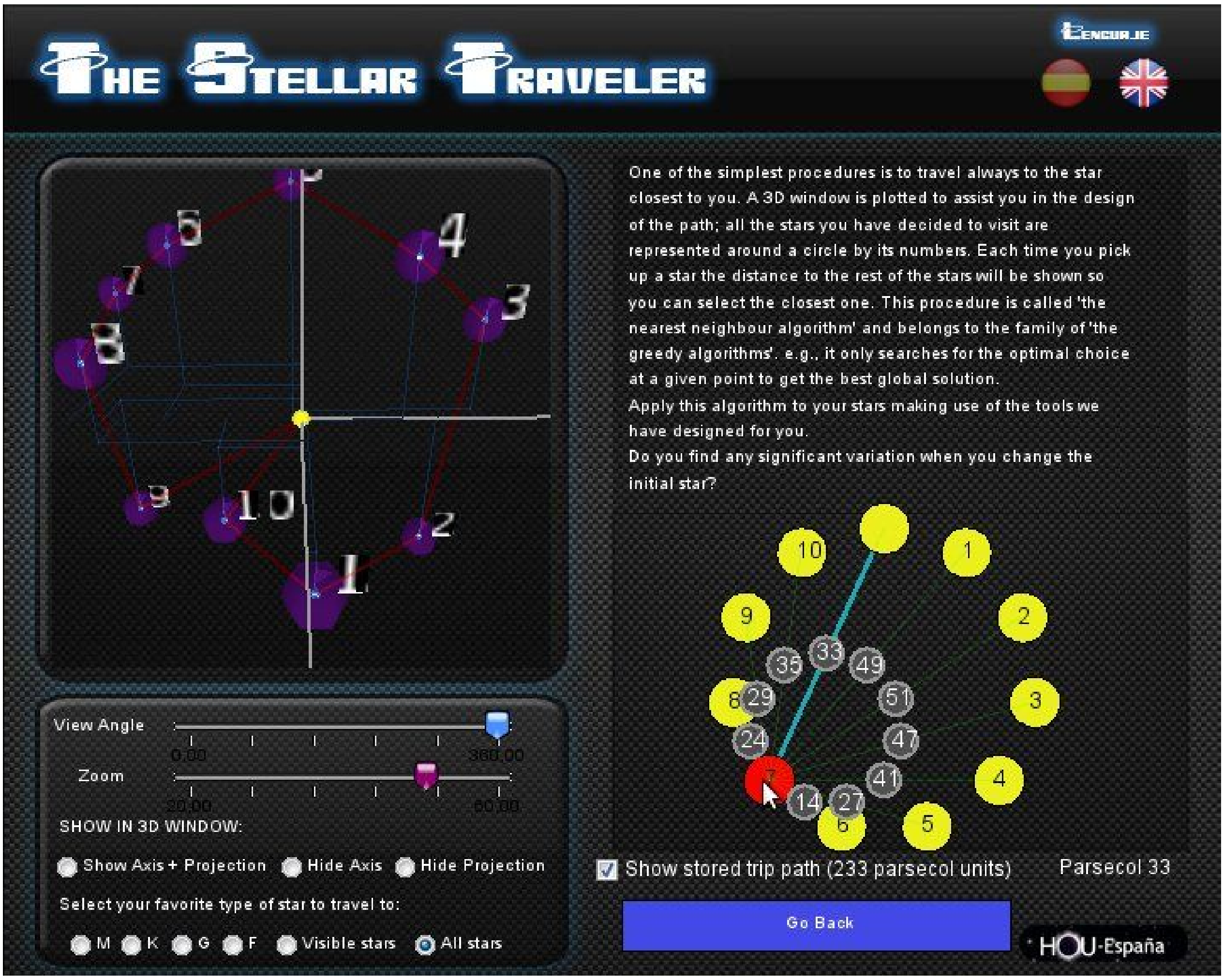

Figure 5: The tool allows to select the nearest neighbor and keeps the accounting on the distance covered in this path.

After the path is completed the students will find that for some selections, this is not the optimal path.

3. Creation

Students will discuss on these results and propose strategies to guarantee that the optimal path is found. New algorithms must be proposed by the teams.

4. Discussion

a. explanation based on evidence

Ask students to answer the following questions:

- why the nearest neighbor algorithm is not optimal always?
- has it anything to do with the clustering in the 3D space of the planetary systems selected?
- has it anything to do with the distance to the Sun?
- which other algorithms would you propose?

The students, grouped in teams, must provide a written report with the answers to these questions.

b. consider other explanations

Discuss with your students about:

- how the nearest neighbor algorithm relates with the permutation problem.
- the algorithms they propose and analyze with them the relevance of clustering on the goodness of the proposed heuristics.

5. Reflection

a. communicate explanation

Make an overview of what has been discussed in the classroom during the exercise.

Compare the analysis made on this problem with other similar problems in everyday life (for instance decoding a password).

Participating roles

In this scenario students start by learning about reference systems and travelling to planetary systems to end discussing about an open, abstract mathematical problem.

The teacher is a facilitator and guides the students through the process of measuring and reflection. He/she introduces students to the pertinent concepts, directs them to the problem at hand by asking questions and shows them how to use the tool. He/she also may introduce students in simple computer programing to try their algorithms and even, may a small competition for the fastest algorithm.

Tools, services and resources

1. Computers with Internet connection and Java
2. Wiki-HOU (Spanish) platform
3. Manuals to assist the teacher accessible through the wiki-HOU:
   - Manual 1: Astronomical coordinates, Distances, Magnitude
   - Manual 2: Heuristics